Quantum Vacuum Fluctuations

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A vacuum fluctuation


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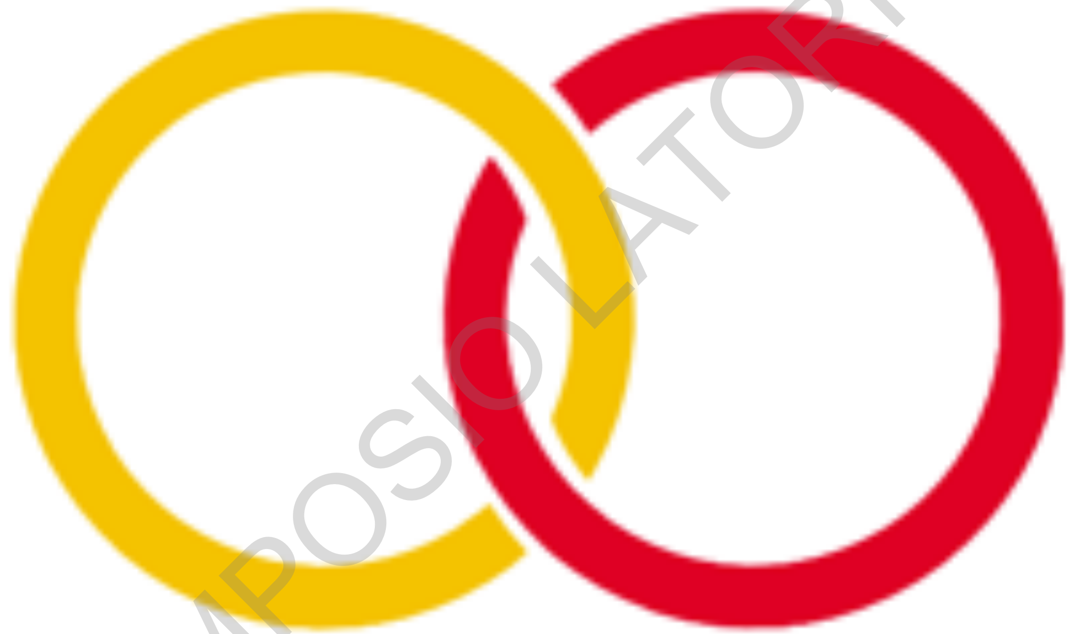
Theory of Relativity

1905

Quantum Theory

1925



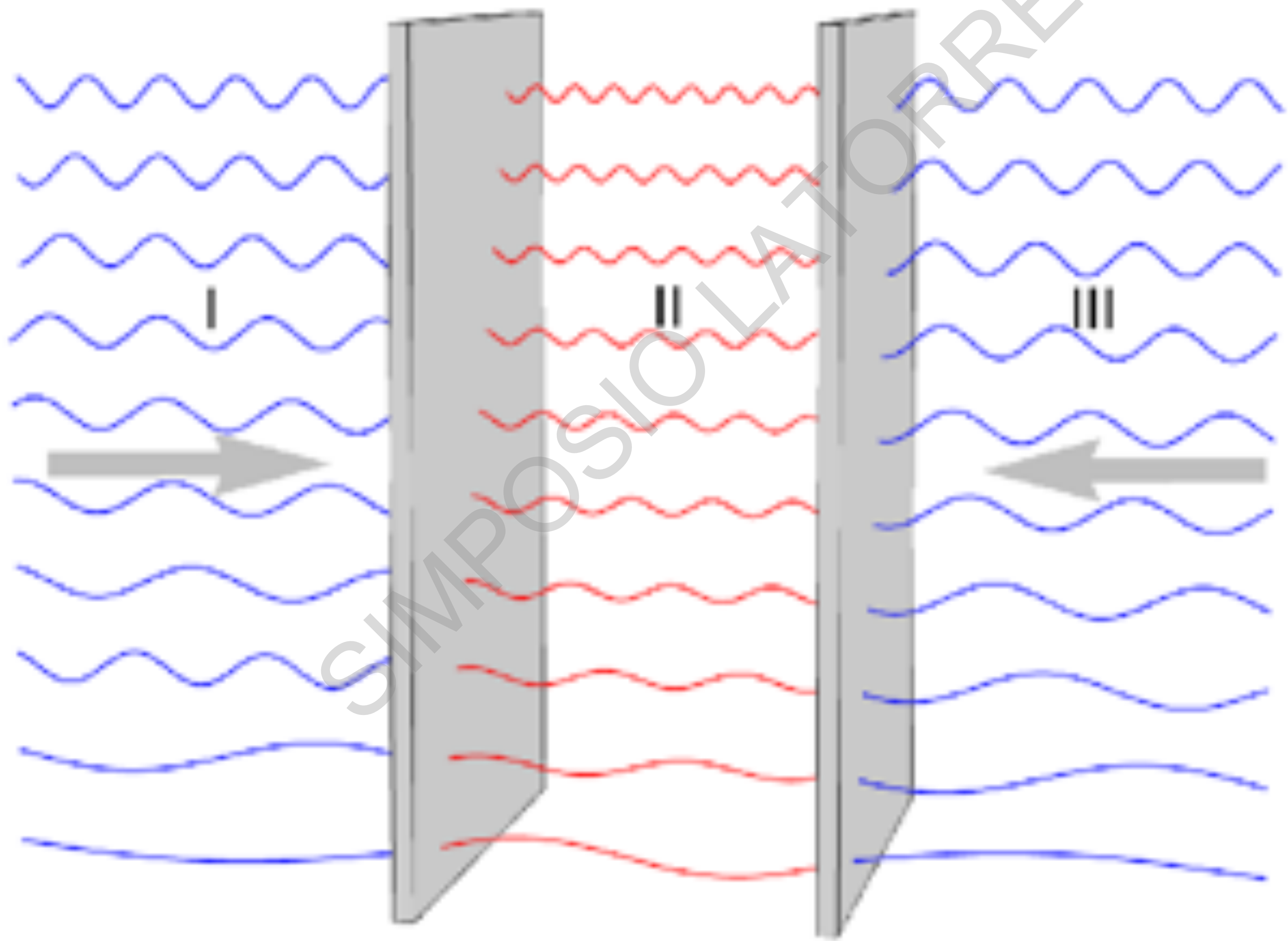
Quantum Field Theory

1948

Vacuum fluctuations

- are real
- do matter
- do weight
- cosmological puzzle

Casimir effect



Mathematics. — *On the attraction between two perfectly conducting plates.* By H. B. G. CASIMIR.

(Communicated at the meeting of May 29, 1948.)

The higher derivatives will contain powers of (π/ak_m). Thus we find

$$\delta E/L^2 = -\hbar c \frac{\pi^2}{24 \times 30} \cdot \frac{1}{a^3}$$

a formula which holds as long as $ak_m \gg 1$. For the force per cm^2 we find

$$F = \hbar c \frac{\pi^2}{240} \frac{1}{a^4} = 0.013 \frac{1}{a_\mu^4} \text{ dyne/cm}^2$$

where a_μ is the distance measured in microns.

We are thus led to the following conclusions. There exists an attractive force between two metal plates which is independent of the material of the plates as long as the distance is so large that for wave lengths comparable with that distance the penetration depth is small compared with the distance. This force may be interpreted as a zero point pressure of electromagnetic waves.

Although the effect is small, an experimental confirmation seems not unfeasable and might be of a certain interest.

Natuurkundig Laboratorium der N.V. Philips' Gloeilampenfabrieken. Eindhoven.)

$$F = -\hbar c \frac{\pi^2}{240} \frac{1}{a^4} = -0.013 \frac{1}{a_\mu^4} \text{ dyne/cm}^2$$

- The formula only depends on h, c and a
- The minus sign means that F it is attractive
- The force is the dominant force between neutral objects a submicron distances
- At a=10nm the Casimir force equals the atmospheric pressure

Origin of Casimir force

- Modification of vacuum (zero-point) energy due to the presence of the plates

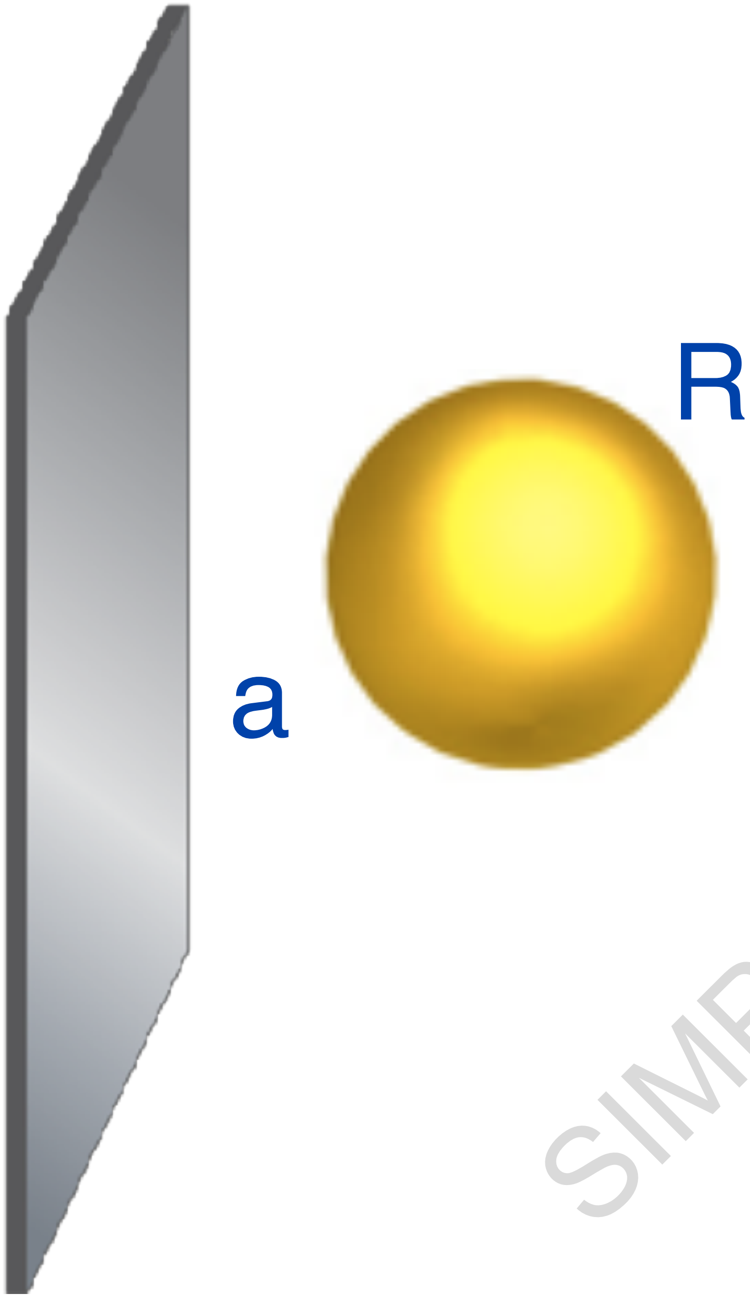
$$\frac{\Delta E}{A} = \sum_{\omega_c} \hbar\omega_c - \sum_{\omega_0} \hbar\omega_0$$

$$\frac{\Delta E}{A} = \sum_{\omega_c} \hbar\,\omega_c\, e^{-\frac{\omega_c}{\omega_\infty}} - \sum_{\omega_0} \hbar\,\omega_0\, e^{-\frac{\omega_0}{\omega_\infty}}$$

$$\omega_0 = \sqrt{k_x^2 + k_y^2 + k_z^2} \qquad \omega_c = \sqrt{k_x^2 + k_y^2 + \left(\frac{\pi n c}{d}\right)^2}$$

$$\frac{\Delta E_c}{A} = f_s\,\hbar\,c\,\omega_\infty^2 - \frac{\hbar\,c\,\pi^2}{720\,a^3}$$

Casimir effect plane-sphere



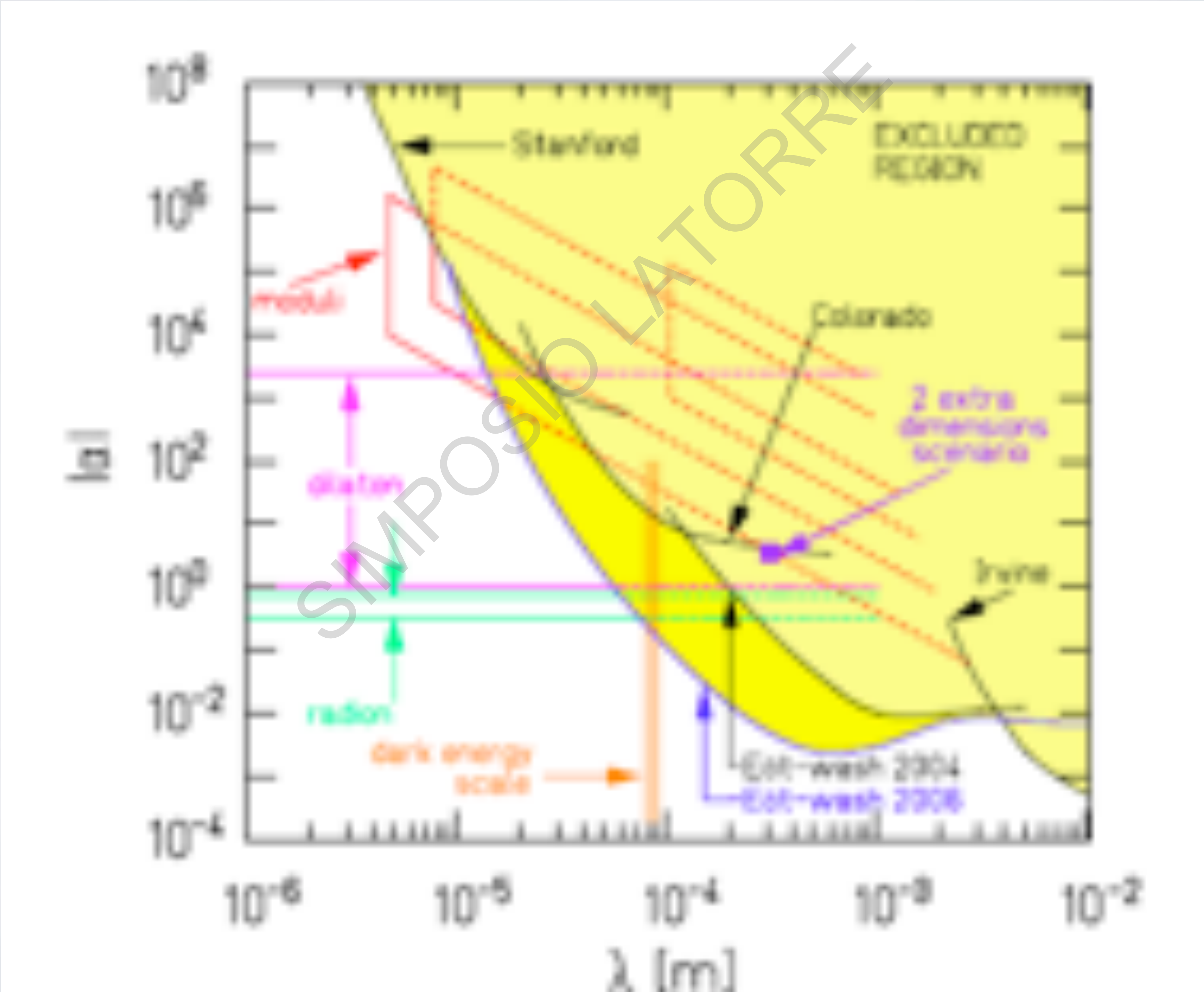
$$\Delta E_c = -\frac{\hbar\, c\, R\, \pi^2}{360\, a^2}$$

Casimir effect experiments

Year	Geometry	Range (μm)	Accuracy (%)	Reference
1958	Plane–plane	$0.3 \div 2.5$	100	Sparnaay 1958
1978	Plane–sphere	$0.13 \div 0.67$	25	van Blokland and Oveerbeek 1978
1997	Plane–sphere	$0.6 \div 12.3$	5	Lamoreaux 1997
1998	Plane–sphere	$0.1 \div 0.9$	1	Mohideen and Roy 1998
2000	Crossed cylinders	$0.02 \div 0.1$	1	Ederth 2000
2001	Plane–sphere	$0.08 \div 1.0$	1	Chan *et al* 2001
2002	Plane–plane	$0.5 \div 3.0$	15	Bressi *et al* 2002
2003	Plane–sphere	$0.2 \div 2.0$	1	Decca *et al* 2003
2018	Sphere-sphere	0.02 - 0.4	1	Munday *et al.* 2018

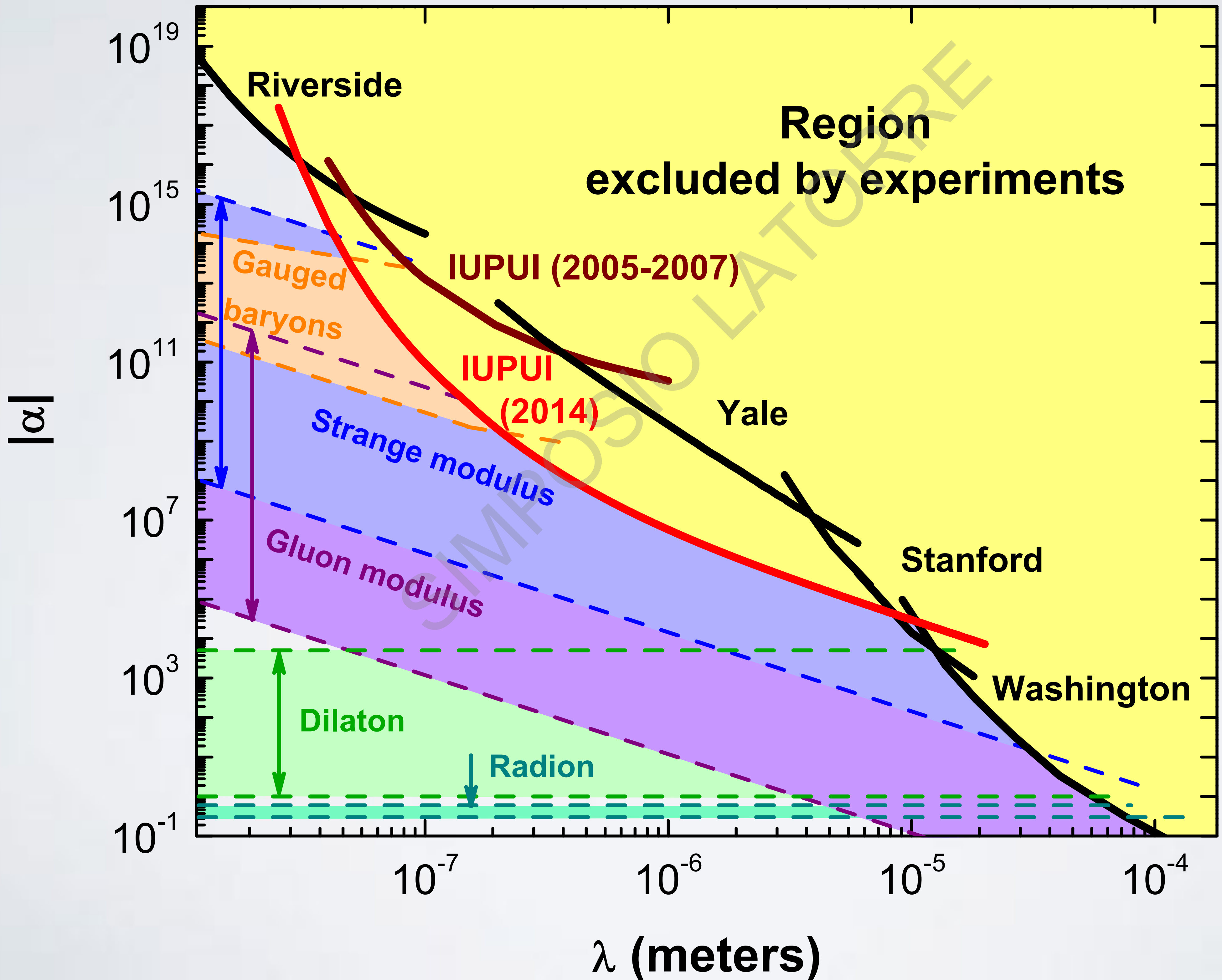
Gravity at short distances

$$V(r) = -G\frac{m_1 m_2}{r}(1 + \alpha e^{-r/\lambda})$$



Experimental bounds

2 orders of magnitude improvement (2014)



The weight of quantum vacuum

$$g_{00} = 1 - g\,z, \quad g_{ij} = -\delta_{ij}$$

$$\frac{\Delta E_c}{A} = -\frac{\hbar\, c\, \pi^2}{720\, a^3}\left(1 + \frac{5}{2}\frac{g\, a}{c^2}\right)$$

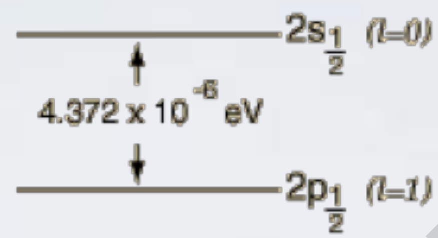
$$\Delta P_c^{\pm} = -\frac{\hbar\, c\, \pi^2}{240\, a^4}\left(1 \pm \frac{2}{3}\frac{g\, a}{c^2}\right)$$

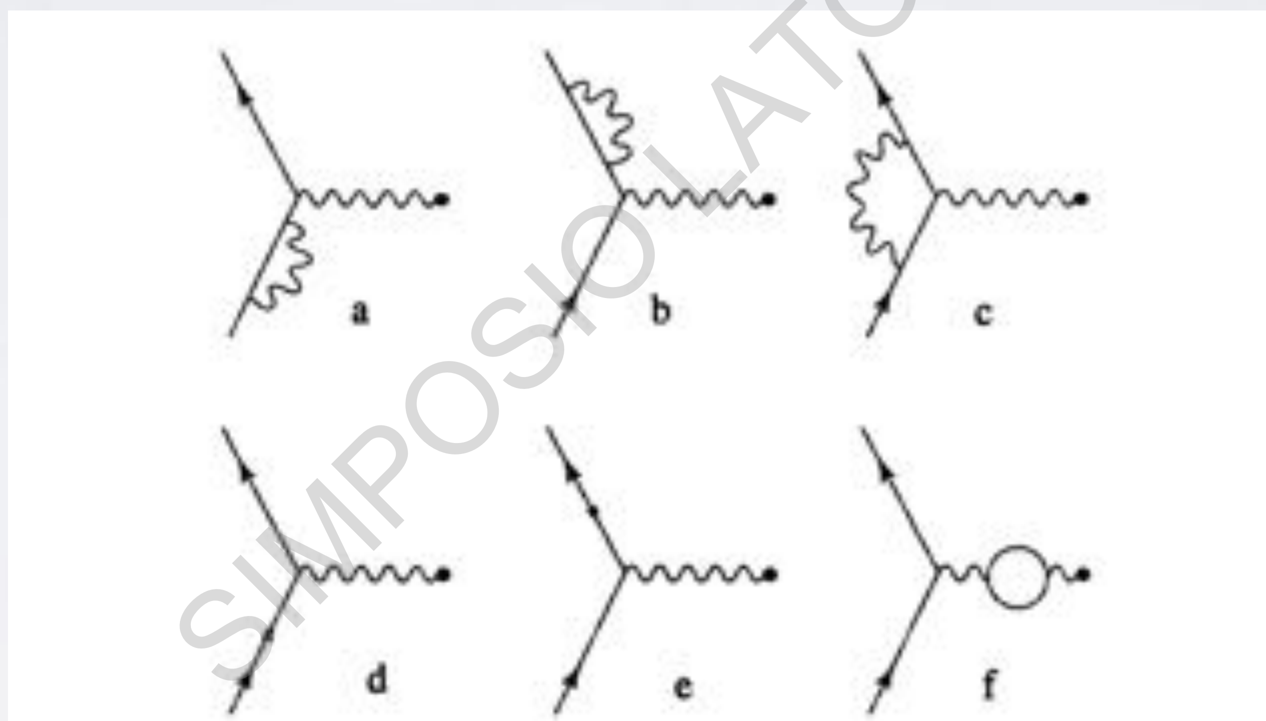
Equivalence principle

$$F_g = -g\,\frac{E_c}{c^2}$$

Lamb shift

1947





Experiment

1057.862 ± 0.020 MHz .

Theory

1057.864±0.014 MHz

Weight of fluctuations. Equivalence principle

Vacuum Energy vs Dark Energy

- Cosmological constant (w=-1)

$$S = \frac{1}{16\,\pi\,G}\int d^4x\sqrt{-g}(R - 2\Lambda_0)$$

$$\frac{E_0}{V} = \frac{1}{8\,\pi\,G}\Lambda_0 = -\frac{P_0}{V}$$

- Cosmological constant is much smaller than any QFT vacuum energy

$$\frac{E_0}{V}^{\text{obs}} \sim (10^{-12}\,\text{GeV})^4$$

$$\frac{E_0}{V}^{(\text{EW})} \sim (100\,\text{GeV})^4 \qquad \frac{E_0}{V}^{(\text{PL})} \sim (10^{18}\,\text{GeV})^4$$

Vacuum Energy vs Dark Energy

- Minkowski spacetime

$$\frac{E_0}{V} = \frac{\hbar\, c}{8\,\pi^2}\omega_\infty^4$$

$$\frac{P_0}{V} = \frac{\hbar\, c}{24\,\pi^2}\omega_\infty^4 \qquad w = \frac{1}{3}$$

- Cosmological background FLRW

$$\frac{E_0}{V} = \frac{\hbar\, c}{8\,\pi^2}\omega_\infty^4 + \frac{H^2(t)}{8\,\pi^2}\omega_\infty^2 + \mathcal{O}(H^4 \log \omega_\infty)$$

$$\frac{P_0}{V} = \frac{\hbar\, c}{24\,\pi^2}\omega_\infty^4 - \frac{H^2(t)}{24\,\pi^2}\omega_\infty^2 + \mathcal{O}(H^4 \log \omega_\infty)$$

Casimir Energy vs Dark Energy

- Casimir Energy density

$$\mathcal{E}_c = \frac{E_c}{A} = -\frac{\hbar\, c\, \pi^2}{720\, a^3}$$

- Casimir Pressure

$$P_c = -\frac{\hbar\, c\, \pi^2}{240\, a^4}$$

- Amazing equation of state

$$\mathcal{E}_c = \frac{1}{3}\, P_c$$

The weight of quantum vacuum

$$\langle T_{\mu\nu} \rangle = -\frac{\hbar\, c\, \pi^2}{720\, a^4} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix}$$

Conformal Invariance

$$\langle T_{\mu\nu} \rangle \neq -\frac{\hbar\, c\, \pi^2}{\lambda a^4} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

There is life beyond QED

Non Abelian gauge theories

Perturbation theory

$D = 3 + 1$

$$\frac{E_c}{A} = -\frac{\hbar\, c\, \pi^2}{720\, a^2}(N^2 - 1)$$

$D = 2 + 1$

$$\frac{E_c}{A} = -\frac{\hbar\, c\, \zeta(3)}{8\, \pi\, a^2}(N^2 - 1)$$

Non Abelian gauge theories

Non-perturbative

$$\hat{H} = \int d^3x \left[-\frac{g^2}{2} \frac{\delta^2}{\delta A^2} + \frac{1}{2g^2} F_{ij} F^{ij} \right]$$

Gauss law

$$\nabla_A \cdot \frac{\delta}{\delta A} \psi(A) = 0 \qquad \psi(A^\Phi) = \psi(A)$$

In 2+1D: holomophic parametrization

$$A_z = \partial_z M M^{-1}$$

Non Abelian gauge theories

Gauge transformations

$$A^\chi = \chi^{-1} A \chi + \chi^{-1} d\chi \qquad M^\chi = \chi M$$

Gauge invariant observables

$$H = M^\dagger M = e^{\tau^a \varphi_a}$$

$$\hat{H} = \frac{1}{2} \int d^3x \, \left[-\frac{\delta^2}{\delta \phi^2} + \phi(-\Delta + m^2)\phi \right] + \cdots$$

$$\phi^a = g \sqrt{\Delta} \, \varphi^a \qquad m = \frac{g^2}{2\pi} c_A \qquad \sigma_R = \frac{m^2 \pi \, c_R}{2 \, c_A}$$

[Karabali-Nair]

Casimir Energy

Dirichlet/Neumann boundary conditions

$$\mathcal{E}(a) = -\frac{1}{8\pi a^2}(2\,m\,a+1)\,e^{-2\,m\,a}$$

In agreement with lattice results

Periodic boundary conditions

$$\mathcal{E}(a) = -\frac{1}{\pi a^2}(2\,m\,a+1)\,e^{-\,m\,a}$$

[M.A. -C. Iuliano]

Casimir Energy

Zaremba boundary conditions

$$\mathcal{E}(a) = -\frac{\hbar\, c}{8\pi a^2}(2\, m\, a + 1)\, e^{-2\, m\, a}$$

Generic boundary conditions

$$\begin{pmatrix} \phi(0) + i\dot{\phi}(0) \\ \phi(a) + i\dot{\phi}(a) \end{pmatrix} = U \begin{pmatrix} \phi(0) - i\dot{\phi}(0) \\ \phi(a) - i\dot{\phi}(a) \end{pmatrix}$$

$$\mathcal{E}(a) = -\frac{\hbar\, c}{8\pi a^2}(2 c_1\, m\, a + c_2)\, e^{-\, m\, a}, \qquad \mathrm{tr}\, \sigma_1 U \neq 0$$

$$\mathcal{E}(a) = -\frac{\hbar\, c}{8\pi a^2}(2\, b_1\, m\, a + b_2)\, e^{-2\, m\, a}, \qquad \mathrm{tr}\, \sigma_1 U = 0$$

[M.A. -C. Iuliano]


Congratulations


There is life beyond 60

Gribov's Quark Confinement

Heavy quark potential:

$$S_E^{YM}(A) = -\frac{1}{2g_s^2}\int d^4x\, Tr(F^{\mu\nu}F_{\mu\nu}) + Q\int dx^0\, A_0^3$$

Solution of motion equations (Coulomb)

$$(A_0)^3(\vec{x}) = \frac{i\alpha}{|\vec{x}-L\vec{e}_3|} - \frac{i\alpha}{|\vec{x}+L\vec{e}_3|}, \quad \alpha = \frac{g_s^2 Q}{4\pi},$$

Instability of Euclidean functional integral

$$\delta^{(2)}S = -\int d^4x\, Tr\left(\tau^\mu(-\delta_{\mu\nu}D^2 + D_\mu D_\nu - 2[F_{\mu\nu}.\cdot])\tau^\nu\right),$$

Quark-Antiquark: Meson

Coulomb potential

$$(A_0)^3(\vec{x}) = \frac{i\alpha}{|\vec{x} - L\vec{e}_3|} - \frac{i\alpha}{|\vec{x} + L\vec{e}_3|},$$

Unstable magnetic modes

$$\vec{\tau}(\vec{x}) = \frac{\vec{x} \times \vec{e}_3}{\rho} \phi(\rho, z)\, \mathsf{T}_{12}, \qquad \tau_0 = 0 \qquad (m = 1),$$

$$\left[\frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial z^2} - \frac{3}{4\rho^2} + \left(\frac{\alpha}{\sqrt{\rho^2 + (z-L)^2}} - \frac{\alpha}{\sqrt{\rho^2 + (z+L)^2}}\right)^2\right]\phi(\rho, z)$$

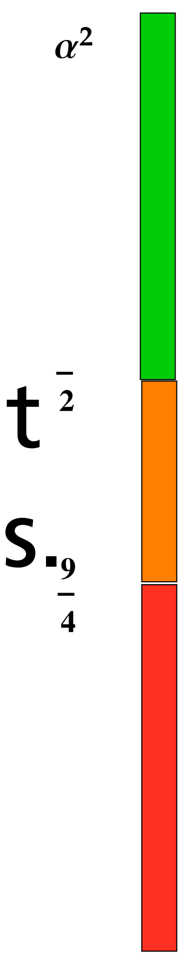
$$= \lambda^2 \phi(\rho, z)$$

Quark-Antiquark Coulomb Instability

- Zero modes with $j = 0$ (pure gauge modes)

- Three different unstable regimes $j = 1$

  - i) $\alpha^2 < 2 \Rightarrow$ no negative eigenvalues
    (Stability of Coulomb potential)
  - ii) $2 < \alpha^2 < \frac{9}{4} \Rightarrow$ two negative eigenvalues at large distances and none at short distances.
    (Instability of Coulomb potential)
  - iii) $\alpha^2 > \frac{9}{4} \Rightarrow \infty$-negative eigenvalues
    (Instability of Coulomb potential)



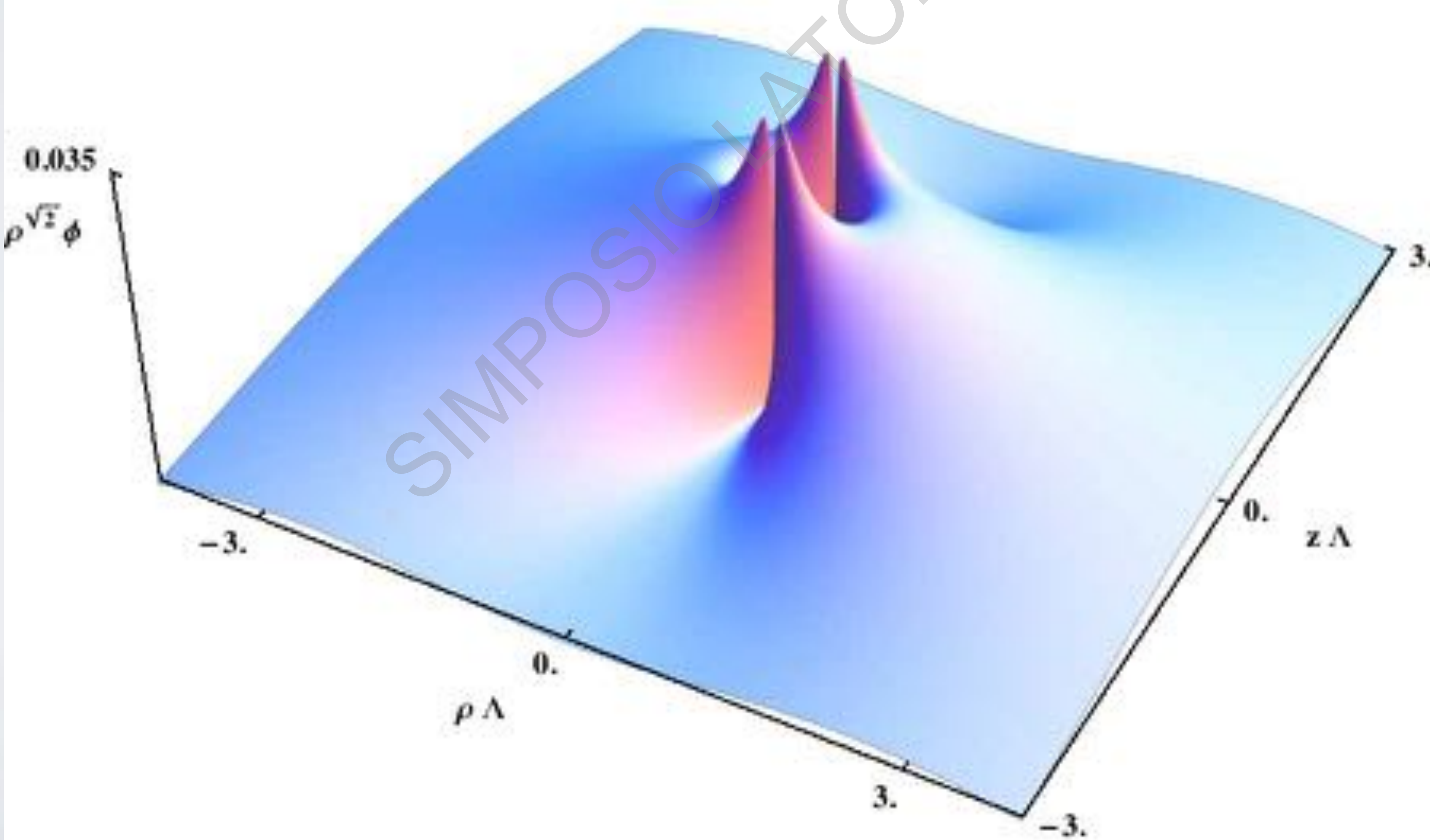
Broken Conformal Symmetry Λ

M.A. & A. Santagata

Unstable solutions

$$2 < \alpha^2 < \frac{9}{4}$$

Symmetric solution (Thick string)

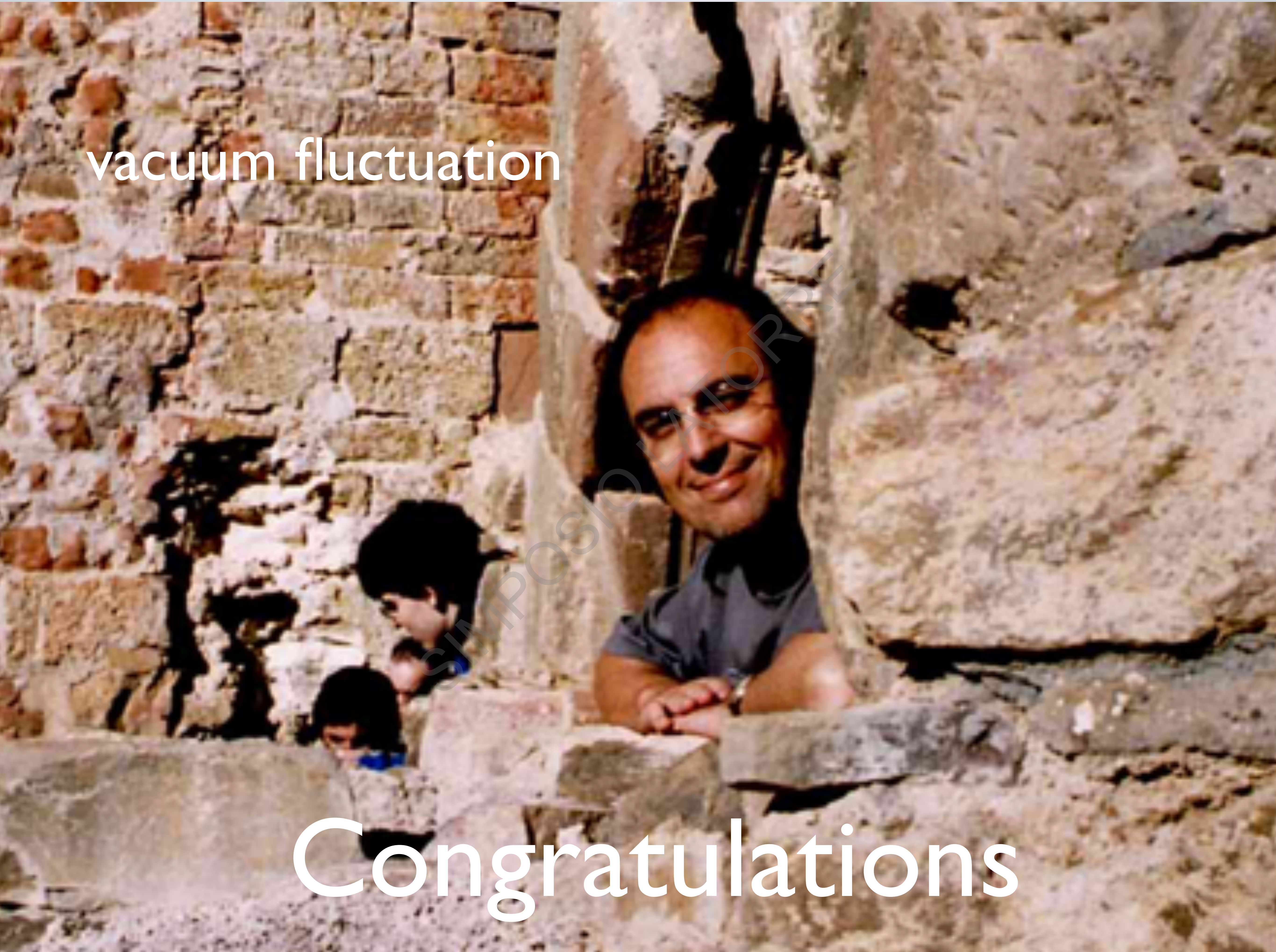


COULOMB PHASE INSTABILITIES

- Gribov picture of confinement derived from first principles
- Weak Coupling regime $\alpha^2 < 2$:
  Coulomb phase is stable (perturbative regime)
- Strong Coupling regime $\alpha^2 > \frac{9}{4}$:
  Coulomb phase is unstable (confinement)
- Intermediate regime $2 < \alpha^2 < \frac{9}{4}$:
  there is a critical quark distance L_c

  $L<L_c$ Coulomb phase stable (asymt. freedom)

  $L>L_c$ Coulomb phase unstable (confinement)


vacuum fluctuation
Congratulations